## Exercises- Predicate Logic

**Exercise 1**

Transform the following sentences from natural language into predicate formulas.

Explain the syntactic elements used in the predicate formulas: variables, constants,

functions symbols, predicate symbols.

1. In a plane if a line  is perpendicular to a constant line  then
2. all the lines parallel to  are perpendicular to .
3. For every positive integer , if  is not a prime, then

there exists a prime  such that  divides  and  is less than .

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

1. D(domain) = P – the plane(the set of all lines)

d, x, y ∈ D, d - constant , x, y - variables

Binary predicate symbols:

per : D x D -> {T, F}, per(x,y) = T if x⊥y

par: D x D -> {T, F}, par(x, y) = T if x || y

par is reflexive, symmetric and transitive

(ꓯx)par(x,x)

(ꓯx) (ꓯy) (par(x, y) -> par(y, x))

(ꓯx) (ꓯy)(ꓯz) ((par(x,y) ^ par(y, z)) -> par(x,z)

per is symmetric

(ꓯx) (ꓯy) (per(x,y)->per(y,z))

Predicate formula:

(ꓯ x)[per(x,d) -> (ꓯy) (par(y, x) -> per(y, d))]

**Exercise 2**

Transform the following statements from natural language into predicate formulas

choosing the appropriate constants, function symbols and predicate symbols:

7. If Santa has some reindeer with a red nose, then every child loves Santa.

10. Caterpillars and snails are much smaller than birds, which are much smaller than foxes,

which in turn are much smaller than wolves.

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

**Exercise 3**

Using the given interpretations evaluate the following formula:

U1= (ꓱx)(A(x) ^ (ꓱx)(B(x)-> (ꓯx)(A(x) V B(x))

Interpretation , where:  the set of all straight lines of a plane 

Let , a constant straight line belonging to the interpretation domain

””;

””;

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U1)=

**Exercise 4**

Choose an arbitrary interpretation with a finite domain for the formula U1 and prove

that it is a model of U1.

U1= (ꓯx)(A(x) <-> B(x)) ->((ꓯx)A(x) <->(ꓯx) B(x))

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U1)=

(ꓯx)x in {5,8,9} B(x) **≡ B(5) ^ B(8)^ B(9)**

(ꓱx)x in {5,8,9} B(x) **≡ B(5) v B(8) v B(9)**

Exercise 5. Succession to the British throne

|  |  |
| --- | --- |
| *Hypotheses*:  . If  is the king and  is his oldest son,  then  can become the king.  . If  is the king and  defeats ,  then  will become the king.  .  is the king.  . defeated .  .  is ’s oldest son.  *Conclusion*:  . Can  become the king? | variables:  constants:  predicate symbols: **unary**: ,  **binary**: , s |

Check whether the conclusion  is derivable from the set of hypotheses {}

using the definition of deduction and the appropriate inference rules.

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

**Exercise 8**

Prove that the following formulas are not valid by finding anti-models for them.

1. ;
2. ;
3. ;
4. ;

V ^ ->   Ꞁ   (ꓱx) (ꓯx) **|= |- ≡**

vI(U1)=